Artham
Resource Material

For Revised Syllabus Session 2024-25

EXEMPLAR SOLUTIONS

MATHS

Class 11

Powered by SCHOOL OF EDUCATORS (SOE)

Chapter 3-
Trigonometric Functions

EXERCISE
SHORT ANSWER TYPE

1. Prove that

$$\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$$

Solution:

According to the question,

$$\text{LHS} = \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{1}{\cos A} - 1}{\frac{\sin A}{\cos A} - \frac{1}{\cos A} + 1}$$

$$= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A}$$

$$= \frac{\sin A + (1 - \cos A)}{\sin A - (1 - \cos A)}$$

Using the identity,

$\sin^2 A + \cos^2 A = 1$, we get,

$\sin A + (1 - \cos A)$.

$$\therefore \text{LHS} = \frac{\sin A + (1 - \cos A)}{\sin A - (1 - \cos A)} \times \frac{\sin A + (1 - \cos A)}{\sin A + (1 - \cos A)}$$

$$= \frac{\{\sin A + (1 - \cos A)\}^2}{\sin^2 A - (1 - \cos A)^2}$$

$$= \frac{\sin^2 A + (1 - \cos A)^2 + 2\sin A\,(1 - \cos A)}{\sin^2 A - (1 - \cos A)^2}$$

$$= \frac{(\sin^2 A + \cos^2 A) + 1 - 2\cos A + 2\sin A\,(1 - \cos A)}{\sin^2 A - \{1 + \cos^2 A - 2\cos A\}}$$

$$= \frac{(1) + 1 - 2\cos A + 2\sin A\,(1 - \cos A)}{(\sin^2 A - 1) - \cos^2 A + 2\cos A}$$

$$= \frac{2(1 - \cos A) + 2\sin A\,(1 - \cos A)}{(-\cos^2 A) - \cos^2 A + 2\cos A}$$

$$= \frac{2(1 + \sin A)(1 - \cos A)}{-2\cos^2 A + 2\cos A}$$

$$= \frac{2(1 + \sin A)(1 - \cos A)}{2\cos A\,(1 - \cos A)}$$

$$= \frac{(1 + \sin A)}{\cos A} = \text{RHS}$$

Hence, L.H.S = R.H.S

2. If [2sinα / (1+cosα+sinα)] = y, then prove that [(1– cosα+sinα) / (1+sinα)] is also equal to y.

$$\left[\text{Hint : Express } \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}.\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}\right]$$

Solution:

According to the question,

y =2sinα /(1+cosα+sinα)

Multiplying numerator and denominator by (1 – cos α + sin α),

We get,

$$\Rightarrow y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} \times \frac{1-\cos\alpha+\sin\alpha}{1-\cos\alpha+\sin\alpha}$$

$$= \frac{2\sin\alpha}{(1+\sin\alpha)+\cos\alpha} \times \frac{(1+\sin\alpha)-\cos\alpha}{(1+\sin\alpha)-\cos\alpha}$$

Using $(a + b)(a-b) = a^2 - b^2$, we get:

$$= \frac{2\sin\alpha\{(1+\sin\alpha)-\cos\alpha\}}{(1+\sin\alpha)^2-\cos^2\alpha}$$

$$= \frac{2\sin\alpha(1+\sin\alpha)-2\sin\alpha\cos\alpha}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$$

Since, $1 - \cos^2\alpha = \sin^2\alpha$

$$\therefore y = \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{\sin^2\alpha+2\sin\alpha+\sin^2\alpha}$$

$$= \frac{2\sin\alpha(1+\sin\alpha-\cos\alpha)}{2\sin\alpha(1+\sin\alpha)}$$

$$\Rightarrow y = \frac{(1+\sin\alpha-\cos\alpha)}{(1+\sin\alpha)}$$

$$\Rightarrow y = \frac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)}$$

Hence Proved

3. If m sin θ = n sin (θ + 2α), then prove that
tan (θ + α) cot α = (m + n)/(m – n)
[Hints: Express sin(θ + 2α) / sinθ = m/n and apply componendo and dividend]
Solution:

According to the question,

m sin θ = n sin (θ + 2α)

To prove:

tan (θ + α)cot α =(m + n)/(m – n)

Proof:

m sin θ = n sin (θ + 2α)

⇒ sin(θ + 2α) / sinθ = m/n

Applying componendo-dividendo rule, we have,

$$\Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{m+n}{m-n}$$

By transformation formula of T-ratios,

We know that,

sin A + sin B = 2 sin ((A+B)/2) cos ((A – B)/2)

And,

sin A – sin B = 2 cos ((A+B)/2) sin ((A – B)/2)

On applying the formula, we get,

$$\frac{2\sin\left(\frac{2\theta+2\alpha}{2}\right)\cos\left(\frac{\theta+2\alpha-\theta}{2}\right)}{2\cos\left(\frac{2\theta+2\alpha}{2}\right)\sin\left(\frac{\theta+2\alpha-\theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\sin(\theta+\alpha)\cos(\alpha)}{\cos(\theta+\alpha)\sin(\alpha)} = \frac{m+n}{m-n}$$

{∵ tan x = (sin x)/(cos x)}

$$\Rightarrow \tan(\theta+\alpha)\cot\alpha = \frac{m+n}{m-n}$$

Therefore, tan (θ + α) cot α = (m + n)/(m – n)

Hence Proved

4. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, **where α lie between 0 and π/4, find value of tan 2α**

[Hint: Express tan 2α as tan (α + β + α – β]

Solution:

According to the question,

cos(α + β) = 4/5 …(i)

We know that,

sin x = √(1 – $\cos^2$x)

Therefore,

sin (α + β) = √(1 – $\cos^2$(α + β))

⇒ sin (α + β) = √(1 – $(4/5)^2$) = 3/5 …(ii)

Also,

sin(α - β) = 5/13 {given} …(iii)

we know that,

cos x = √(1 – $\sin^2$x)

Therefore,

cos (α - β) = √(1 – $\sin^2$(α - β))

⇒ cos (α - β) = √(1 – $(5/13)^2$) = 12/13 …(iv)

Therefore,

tan 2α = tan (α + β + α – β)

We know that,

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\therefore \tan 2\alpha = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$\Rightarrow \tan 2\alpha = \frac{\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} + \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}{1 - \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \times \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}$$

From equation i, ii, iii and iv we have,

$$\Rightarrow \tan 2\alpha = \frac{\frac{\frac{3}{5}}{\frac{4}{5}} + \frac{\frac{5}{13}}{\frac{12}{13}}}{1 - \frac{\frac{3}{5}}{\frac{4}{5}} \times \frac{\frac{5}{13}}{\frac{12}{13}}}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}}$$

$$= \frac{\frac{9+5}{12}}{1 - \frac{15}{48}}$$

$$\Rightarrow \tan 2\alpha = \frac{14}{12\left(\frac{33}{48}\right)}$$

$$= \frac{56}{33}$$

Hence, $\tan 2\alpha = 56/33$

5. If tanx = b/a then find the value of

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}.$$

Solution:

According to the question,

$\tan x = b/a$

Let,

$$y = \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$\therefore y = \sqrt{\frac{a\left(1+\frac{b}{a}\right)}{a\left(1-\frac{b}{a}\right)}} + \sqrt{\frac{a\left(1-\frac{b}{a}\right)}{a\left(1+\frac{b}{a}\right)}}$$

$$= \sqrt{\frac{(1+\tan x)}{(1-\tan x)}} + \sqrt{\frac{(1-\tan x)}{(1+\tan x)}}$$

$$= \frac{\sqrt{1+\tan x}}{\sqrt{1-\tan x}} + \frac{\sqrt{1-\tan x}}{\sqrt{1+\tan x}}$$

$$= \frac{\left(\sqrt{1+\tan x}\right)^2 + \left(\sqrt{1-\tan x}\right)^2}{\left(\sqrt{1-\tan x}\right)\left(\sqrt{1+\tan x}\right)}$$

$$= \frac{1+\tan x + 1 - \tan x}{\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\tan^2 x}}$$

$$\therefore y = \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2}{\sqrt{1-\tan^2 x}}$$

$$= \frac{2}{\sqrt{1 - \frac{\sin^2\theta}{\cos^2\theta}}}$$

$$= \frac{2}{\frac{\sqrt{\cos^2\theta - \sin^2\theta}}{\cos\theta}}$$

$$\because \cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$= \frac{2\cos\theta}{\sqrt{\cos 2\theta}}$$

6. Prove that cos θ cos θ/2 – cos 3θ cos 9θ/2 = sin7θ sin4θ

[Hint: Express L.H.S. = ½ [2cos θcos θ/2 – 2cos 3θ cos 9θ / 2]

Solution:

Using transformation formula, we get,

2 cos A cos B = cos(A + B) + cos (A – B)

-2 sin A sin B = cos(A + B) - cos (A – B)

Multiplying and dividing the expression by 2.

$\therefore$ LHS $= \frac{1}{2}\left(2\cos\theta\cos\frac{\theta}{2} - 2\cos 3\theta\cos\frac{9\theta}{2}\right)$

Applying transformation formula, we get,

LHS $= \frac{1}{2}\left(\cos\left(\theta + \frac{\theta}{2}\right) + \cos\left(\theta - \frac{\theta}{2}\right) - \left\{\cos\left(3\theta + \frac{9\theta}{2}\right) + \cos\left(3\theta - \frac{9\theta}{2}\right)\right\}\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\left(\frac{15\theta}{2}\right) - \cos\left(-\frac{3\theta}{2}\right)\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right)$ $\{\because \cos(-x) = \cos x\}$

$\Rightarrow$ LHS $= \frac{1}{2}\left(\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(2\sin\left(\frac{\frac{\theta}{2}+\frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{15\theta}{2}-\frac{\theta}{2}}{2}\right)\right)$

$\Rightarrow$ LHS $= \frac{1}{2}\left(2\sin\left(\frac{8\theta}{2}\right)\sin\left(\frac{7\theta}{2}\right)\right)$

$\therefore$ LHS $= \sin 4\theta \sin\left(\frac{7\theta}{2}\right)$ = RHS

Hence,

$\cos\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 4\theta\sin\left(\frac{7\theta}{2}\right)$

7. If a cos θ + b sin θ = m and a sin θ – b cos θ = n, then show that $a^2 + b^2 = m^2 + n^2$.

Solution:

According to the question,

a cos θ + b sin θ = m …(i)

a sin θ – b cos θ = n …(ii)

Squaring and adding equation 1 and 2, we get,

$(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = m^2 + n^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta = m^2 + n^2$

$\Rightarrow a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$

Using, $\sin^2\theta + \cos^2\theta = 1$,

We get,

$\Rightarrow a^2 + b^2 = m^2 + n^2$

8. Find the value of tan 22°30'.

[Hint: Let θ = 45°, use $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\theta}{1+\cos\theta}$]

Solution:

Let, θ = 45°

As we need to find: tan 22°30' = tan (θ/2)

We know that,

sin θ = cos θ = 1/√2 (for θ = 45°)

Since,

$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

Multiplying 2cos θ/2 in numerator and denominator, we get,

$\Rightarrow \tan\frac{\theta}{2} = \frac{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

By applying formula of T-ratios of multiple angles-

$\sin 2x = 2\sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$ or $1 + \cos 2x = 2\cos^2 x$

$\therefore \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

$\Rightarrow \tan 22°30' = \frac{\sin 45°}{1+\cos 45°}$

$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}+1}$

Therefore, $\tan 22°30' = \sqrt{2} - 1$

9. Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

Solution:

$\sin 4A = \sin(2A + 2A)$

We know that,

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

Therefore, $\sin 4A = \sin 2A\cos 2A + \cos 2A\sin 2A$

$\Rightarrow \sin 4A = 2\sin 2A\cos 2A$

From T-ratios of multiple angle,

We get,

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$

$\Rightarrow \sin 4A = 2(2\sin A\cos A)(\cos^2 A - \sin^2 A)$

$\Rightarrow \sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$

Hence, $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$

10. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$

[Hint: $m + n = 2\tan\theta$, $m - n = 2\sin\theta$, then use $m^2 - n^2 = (m + n)(m - n)$]

Solution:

According to the question,

$\tan\theta + \sin\theta = m$ …(i)

$\tan\theta - \sin\theta = n$ …(ii)

Adding equation i and ii,

$2\tan\theta = m + n$ …(iii)

Subtracting equation ii from i,

We get,

$2\sin\theta = m - n$ …(iv)
Multiplying equations (iii) and (iv),
$2\sin\theta\,(2\tan\theta) = (m + n)(m - n)$
$\Rightarrow 4\sin\theta\tan\theta = m^2 - n^2$
Hence,
$m^2 - n^2 = 4\sin\theta\tan\theta$

11. If tan (A + B) = p, tan (A – B) = q, then show that tan 2A = (p + q) / (1 – pq).
[Hint: Use 2A = (A + B) + (A – B)]
Solution:

We know that,
$\tan 2A = \tan(A + B + A - B)$
And also,
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\therefore \tan 2A = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B)\tan(A-B)}$$
Substituting the values given in question,
$$\Rightarrow \tan 2A = \frac{p+q}{1-pq}$$
$$\text{Hence, } \tan 2A = \frac{p+q}{1-pq}$$

12. If cosα + cosβ = 0 = sinα + sinβ, then prove that cos 2α + cos 2β = –2cos (α + β).
[Hint: cosα + cosβ)^2 – (sinα + sinβ)^2 = 0]
Solution:

According to the question,
$\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ …(i)
Since, LHS $= \cos 2\alpha + \cos 2\beta$
We know that,
$\cos 2x = \cos^2 x - \sin^2 x$
Therefore,
LHS $= \cos^2\alpha - \sin^2\alpha + (\cos^2\beta - \sin^2\beta)$
$\Rightarrow$ LHS $= \cos^2\alpha + \cos^2\beta - (\sin^2\alpha + \sin^2\beta)$
Also, since,
$a^2 + b^2 = (a+b)^2 - 2ab$
$\Rightarrow$ LHS $= (\cos\alpha + \cos\beta)^2 - 2\cos\alpha\cos\beta - (\sin\alpha + \sin\beta)^2 + 2\sin\alpha\sin\beta$
From equation (i),
$\Rightarrow$ LHS $= 0 - 2\cos\alpha\cos\beta - 0 + 2\sin\alpha\sin\beta$
$\Rightarrow$ LHS $= -2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$
$\because \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
Therefore, LHS $= -2\cos(\alpha + \beta) =$ RHS
Hence, $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

13.

If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, **then show that** $\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$.

[Hint: Use componendo and Dividendo]

Solution:

According to the question,

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

Since, sin(A+B) = sin A cos B + cos A sin B

$$\therefore \frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$$

Applying componendo-dividendo rule,

We get,

$$\Rightarrow \frac{(\sin x \cos y + \cos x \sin y) + (\sin x \cos y - \cos x \sin y)}{(\sin x \cos y + \cos x \sin y) - (\sin x \cos y - \cos x \sin y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$$

$$\Rightarrow \frac{2 \sin x \cos y}{2 \cos x \sin y} = \frac{2a}{2b}$$

$$\Rightarrow \left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos y}{\sin y}\right) = \frac{a}{b}$$

Since, tan A = (sinA)/cos(A)

$$\Rightarrow \tan x \left(\frac{1}{\tan y}\right) = \frac{a}{b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$

14.

If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, **then show that sinα + cosα = √2 cos θ.**

[Hint: Express tanθ = tan(α – π/2) θ = α – π/4]

Solution:

We know that,

$$\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

$$\Rightarrow \tan\theta = \frac{\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} - 1\right)}{\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} + 1\right)}$$

Since, tan A = (sin A)/(cos A)

⇒ tan θ = (tan α – 1) / (tan α + 1)

Since, tan π/4 = 1

$$\Rightarrow \tan\theta = \frac{\left(\tan\alpha - \tan\frac{\pi}{4}\right)}{\left(1 + \tan\frac{\pi}{4}.\tan\alpha\right)}$$

We know that,

$\tan(x-y) = (\tan x - \tan y) / (1 + \tan x . \tan y)$

Therefore, $\tan \theta = \tan (\alpha - \pi/4)$

$\Rightarrow \theta = \alpha - \pi/4$

$\Rightarrow \alpha = \theta + \pi/4$ …(i)

To prove,

$\sin\alpha + \cos\alpha = \sqrt{2} \cos \theta$

$\because$ LHS = $\sin\alpha + \cos\alpha$

From equation (i)

$\Rightarrow$ LHS = $\sin(\theta + \pi/4) + \cos(\theta + \pi/4)$

$\because \sin(x + y) = \sin x \cos y + \cos x \sin y$

And, $\cos(x + y) = \cos x \cos y - \sin x \sin y$

Therefore, LHS = $\sin \theta \cos(\pi/4) + \sin(\pi/4)\cos \theta + \cos \theta \cos(\pi/4) - \sin(\pi/4)\sin \theta$

$\because \sin(\pi/4)=\cos(\pi/4) = 1/\sqrt{2}$

$\Rightarrow$ LHS = $\sin \theta (1/\sqrt{2}) + (1/\sqrt{2}) \cos \theta + \cos \theta (1/\sqrt{2}) - \sin \theta (1/\sqrt{2})$

$\Rightarrow$ LHS = $2 \cos \theta (1/\sqrt{2})$

$\Rightarrow$ LHS = $\sqrt{2} \cos \theta$ = RHS

Therefore, $\sin\alpha + \cos\alpha = \sqrt{2} \cos \theta$

15. If $\sin \theta + \cos \theta = 1$, then find the general value of θ.

Solution:

According to the question,

$\sin \theta + \cos \theta = 1$

As, $\sin \theta + \cos \theta = 1$

$\Rightarrow \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right) = 1$

We know that,

$\sin (\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$

$\Rightarrow \sqrt{2}\left(\sin\theta\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\theta\right) = 1$

We know that,

$\sin (A+B) = \sin A\cos B + \cos A\sin B$

$\Rightarrow \sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\frac{\pi}{4} + \theta\right) = \sin\frac{\pi}{4}$

Since we know,

If $\sin \theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$

We get,

$\theta + \pi/4 = n\pi + (-1)^n(\pi/4)$

$\Rightarrow \theta = n\pi + (\pi/4)((-1)^n - 1)$

16. Find the most general value of θ satisfying the equation $\tan \theta = -1$ and $\cos \theta = 1/\sqrt{2}$

Solution:

According to the question,

We have,

tan θ = -1

And cos θ =1/√2 .

⇒ θ = – π/4

So, we know that,

θ lies in IV quadrant.

θ = 2π – π/4 = 7π/4

So, general solution is θ = 7π/4 + 2 n π, n∈ Z

17. If cot θ + tan θ = 2 cosec θ, then find the general value of θ.

Solution:

According to the question,

$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = 2\,\mathrm{cosec}\,\theta$

Since,

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = 2\,\mathrm{cosec}\,\theta$

⇒ 1 = 2 cosec θ sin θ cos θ

We know that,

sin θ cosec θ = 1

⇒ 1 = 2 cos θ

⇒ cos θ = 1/2 = cos(π/3)

Hence,

The solution of cos x = cos α can be given by,

x = 2mπ ± α ∀ m ∈ Z

⇒ θ = 2nπ ± π/3, n ∈ Z

18. If 2sin^{2}θ = 3cos θ, where 0 ≤ θ ≤ 2π, then find the value of θ.

Solution:

According to the question,

2sin^{2}θ = 3cos θ

We know that,

sin^{2}θ = 1 – cos^{2}θ

Given that,

2 sin^2 θ = 3 cos θ

2 – 2 cos^2 θ = 3 cos θ

2 cos^2 θ + 3 cos θ – 2= 0

(cos θ + 2)(2 cos θ – 1) = 0

Therefore,

cos θ = ½ = cos π/3

θ = π/3 or 2π – π/3

θ = π/3, 5π/3

Therefore, 2(1 – cos^{2}θ) = 3cos θ

⇒ 2 – 2cos^{2}θ = 3cos θ

⇒ 2cos^{2}θ + 3cos θ - 2 = 0

⇒ 2cos^{2}θ + 4cos θ - cos θ - 2 = 0

⇒ 2cos θ (cos θ+ 2) +1 (cos θ + 2) = 0
⇒ (2cos θ + 1)(cos θ + 2) = 0
Since, cos θ ∈ [-1,1] , for any value θ.
So, cos θ ≠ - 2
Therefore,
2 cos θ - 1 = 0
⇒ cos θ = ½
= π/3 or 2π – π/3
θ = π/3, 5π/3

19. If sec x cos 5x + 1 = 0, where 0 < x ≤ π/2, then find the value of x.
Solution:

According to the question,
sec x cos 5x = -1
⇒ cos 5x = -1/sec x
We know that,
sec x = 1/cos x
⇒ cos 5x + cos x = 0
By transformation formula of T-ratios,
We know that,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\Rightarrow 2\cos\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) = 0$$

⇒ 2 cos 3x cos 2x = 0
⇒ cos 3x = 0 or cos 2x = 0
∵ 0 < x ≤ π/2
Therefore, 0< 2x ≤ π or 0< 3x ≤ 3π/2
Therefore, 2x = π/2
⇒ x = π/4
3x = π/2
⇒ x = π/6
Or 3x = 3π/2
⇒ x = π/2
Hence, x = π/6, π/4, π/2.

20. If sin (θ + α) = a and sin (θ + β) = b, then prove that cos 2(α – β) – 4ab cos (α – β) = 1 – $2a^2 – 2b^2$
Solution:

According to the question,
sin (θ + α) = a and sin(θ + β) = b
LHS = cos 2(α – β) – 4ab cos (α – β)
Using cos 2x = $2\cos^2x$ – 1,
Let us solve,
⇒ LHS = $2\cos^2(\alpha - \beta)$ - 1 – 4ab cos(α – β)

$\Rightarrow$ LHS = $2\cos(\alpha - \beta)\{\cos(\alpha - \beta) - 2ab\} - 1$

Since,

$\cos(\alpha - \beta) = \cos\{(\theta + \alpha) - (\theta + \beta)\}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\Rightarrow \cos(\alpha - \beta) = \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$

Since,

$\sin(\theta + \alpha) = a$

$\Rightarrow \cos(\theta + \alpha) = \sqrt{(1 - \sin^2(\theta + \alpha)} = \sqrt{(1 - a^2)}$

Similarly,

$\cos(\theta + \beta) = \sqrt{(1 - b^2)}$

Therefore,

$\cos(\alpha - \beta) = \sqrt{(1-a^2)}\sqrt{(1-b^2)} + ab$

Therefore,

LHS = $2\{ab + \sqrt{(1 - a^2)(1 - b^2)}\}\{ab + \sqrt{(1 - a^2)(1 - b^2)} -2ab\} - 1$

$\Rightarrow$ LHS = $2\{\sqrt{(1 - a^2)(1 - b^2)} + ab\}\{\sqrt{(1 - a^2)(1 - b^2)} - ab\}-1$

Using $(x + y)(x - y) = x^2 - y^2$

$\Rightarrow$ LHS = $2\{(1-a^2)(1-b^2) - a^2b^2\} - 1$

$\Rightarrow$ LHS = $2\{1 - a^2 - b^2 + a^2b^2\} - 1$

$\Rightarrow$ LHS = $2 - 2a^2 - 2b^2 - 1$

$\Rightarrow$ LHS = $1 - 2a^2 - 2b^2$ = RHS

Therefore,

We get,

$\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

21. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that $\tan\theta = ((1 - m)/(1 + m))\cot\phi$
[Hint: Express $\cos(\theta + \phi)/\cos(\theta - \phi) = m/l$ and apply Componendo and Dividendo]
Solution:

According to the question,

$\cos(\theta + \phi) = m \cos(\theta - \phi)$

$\because \cos(\theta + \phi) = m \cos(\theta - \phi)$

$\Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{1}{m}$

Applying componendo – dividend, we get,

$\Rightarrow \frac{\cos(\theta-\phi)+\cos(\theta+\phi)}{\cos(\theta-\phi)-\cos(\theta+\phi)} = \frac{1+m}{1-m}$

From transformation formula, we know that,

$\cos(A+B) + \cos(A - B) = 2\cos A\cos B$

$\cos(A - B) - \cos(A + B) = 2\sin A\sin B$

$\Rightarrow \frac{2\cos\theta\cos\phi}{2\sin\theta\sin\phi} = \frac{1+m}{1-m}$

Since, $(\cos\theta)/(\sin\theta) = \cot\theta$

$\Rightarrow \cot\theta\cot\phi = \frac{1+m}{1-m}$

$\Rightarrow \left(\frac{1-m}{1+m}\right)\cot\phi = \frac{1}{\cot\theta}$

$\Rightarrow \tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$

22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

Solution:

According to the question,

Let, $y = 3[\sin^4(3\pi/2 - \alpha) + \sin^4(3\pi + \alpha)] - 2[\sin^6(\pi/2 + \alpha) + \sin^6(5\pi - \alpha)]$

We know that,

$\sin(3\pi/2 - \alpha) = -\cos\alpha$

$\sin(3\pi + \alpha) = -\sin\alpha$

$\sin(\pi/2 + \alpha) = \cos\alpha$

$\sin(5\pi - \alpha) = \sin\alpha$

Therefore,

$y = 3[(-\cos\alpha)^4 + (-\sin\alpha)^4] - 2[\cos^6\alpha + \sin^6\alpha]$

$\Rightarrow y = 3[\cos^4\alpha + \sin^4\alpha] - 2[\sin^6\alpha + \cos^6\alpha]$

$\Rightarrow y = 3[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha)^3 + (\cos^2\alpha)^3]$

Since, we know that,

$\sin^2\alpha + \cos^2\alpha = 1$

Also, we know that,

$a^3+b^3 = (a+b)(a^2 - ab + b^2)$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)(\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha)]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[(\sin^2\alpha + \cos^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$\Rightarrow y = 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha$

$\Rightarrow y = 1$

23. If a cos2θ + b sin 2θ = c has α and β as its roots, then prove that tan α + tan β = 2b/(a + c)
[Hint: Use the identities cos 2θ = ((1 – tan^2 θ)/(1 + tan^2 θ) and sin 2θ = 2tan θ/(1 + tan^2 θ)]
Solution:

According to the question,
a cos2θ + b sin 2θ = c
α and β are the roots of the equation.
Using the formula of multiple angles,
We know that,

$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$\therefore a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - c = 0$

⇒ a(1 – tan^{2}θ) + 2b tan θ - c(1 + tan^{2}θ) = 0
⇒ (-c – a)tan^{2}θ + 2b tan θ - c + a = 0 …(i)
We know that,
The sum of roots of a quadratic equation, ax^2 + bx + c = 0 is given by (-b/a)
Therefore,
tan α + tan β = –2b/–(c + a) = 2b/(c + a)
Hence, tan α + tan β = 2b/(c + a)

24. If x = sec ϕ – tan ϕ and y = cosec ϕ + cot ϕ, then show that xy + x – y + 1 = 0.
[Hint: Find xy + 1 and then show tan x – y = –(xy + 1)]
Solution:

According to the question,
x = sec ϕ – tan ϕ and y = cosec ϕ + cot ϕ
Given that, LHS = xy + x – y + 1

$= (\sec\phi - \tan\phi)(\mathrm{cosec}\,\phi + \cot\phi) + (\sec\phi - \tan\phi) - (\mathrm{cosec}\,\phi + \cot\phi) + 1$

$= \sec\phi\,\mathrm{cosec}\,\phi + \cot\phi\sec\phi - \tan\phi\cot\phi - \tan\phi\,\mathrm{cosec}\,\phi + \sec\phi - \tan\phi - (\mathrm{cosec}\,\phi + \cot\phi) + 1$

$= \frac{1}{\sin\phi\cos\phi} + \frac{1}{\sin\phi} - 1 - \sec\phi + \sec\phi - \tan\phi - \left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right) + 1$

$= \frac{1}{\sin\phi\cos\phi} + \frac{1}{\sin\phi} - \tan\phi - \left(\frac{1}{\sin\phi} + \frac{\cos\phi}{\sin\phi}\right)$

$= \frac{1}{\sin\phi\cos\phi} - \frac{\cos\phi}{\sin\phi} - \frac{\sin\phi}{\cos\phi}$

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{\cos\phi}{\sin\phi} + \frac{\sin\phi}{\cos\phi}\right)$

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{\cos^2\phi + \sin^2\phi}{\sin\phi\cos\phi}\right)$

Since, sin^{2}θ + cos^{2}θ = 1

$= \frac{1}{\sin\phi\cos\phi} - \left(\frac{1}{\sin\phi\cos\phi}\right) = 0$

Thus, LHS = xy + x – y + 1 = 0

25. If θ lies in the first quadrant and cos θ = 8/17, then find the value of cos(30° + θ) + cos (45° – θ) + cos (120° – θ)
Solution:

According to the question,
$\cos\theta = 8/17$
$\sin\theta = \pm\sqrt{(1 - \cos^2\theta)}$
Since, θ lies in first quadrant, only positive sign can be considered.
$\Rightarrow \sin\theta = \sqrt{(1 - 64/289)} = 15/17$
Let, $y = \cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$
We know that,
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
Therefore,
$y = \cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$
Substituting values of cos30°, sin30°, cos 120°, sin120° and cos 45°

$$\Rightarrow y = \frac{\sqrt{3}}{2}.\frac{8}{17} - \frac{1}{2}.\frac{15}{17} + \frac{1}{\sqrt{2}}.\frac{8}{17} + \frac{1}{\sqrt{2}}.\frac{15}{17} + \left(-\frac{1}{2}\right)\left(\frac{8}{17}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{15}{17}\right)$$

$$= \frac{8\sqrt{3}}{34} - \frac{15}{34} + \frac{8+15}{17\sqrt{2}} - \frac{8}{34} + \frac{15\sqrt{3}}{34}$$

$$= \frac{23\sqrt{3}}{34} + \frac{23}{17\sqrt{2}} - \frac{23}{34}$$

$$\Rightarrow y = \frac{23}{34}(\sqrt{3} + \sqrt{2} - 1)$$

26. Find the value of the expression $\cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8)$.
[Hint: Simplify the expression to

$$2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right) = 2\left[\left(\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8}\right)^2 - 2\cos^2\frac{\pi}{8}\cos^2\frac{3\pi}{8}\right]$$

Solution:

According to the question,
Let $y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(5\pi/8) + \cos^4(7\pi/8)$.
$\Rightarrow y = \cos^4(\pi/8) + \cos^4(3\pi/8) + \cos^4(\pi - 3\pi/8) + \cos^4(\pi - \pi/8)$.
Since we know that, $\cos(\pi - x) = -\cos x$, we get,

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{\pi}{8}\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$$
$$= 2\left(\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right)$$
$$= 2\left[\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)^2 - 2\cos^2\frac{\pi}{8}.\sin^2\frac{\pi}{8}\right]$$
$$= 2\left[1 - 2\cos^2\frac{\pi}{8}.\sin^2\frac{\pi}{8}\right]$$
$$= 2 - \left(2\cos\frac{\pi}{8}.\sin\frac{\pi}{8}\right)^2$$
$$= 2 - \left(\sin\frac{2\pi}{8}\right)^2$$
$= 2 – (1/\sqrt{2})^2$
$= 2 – ½$
$= 3/2$

27. Find the general solution of the equation
$5\cos^2\theta + 7\sin^2\theta – 6 = 0$
Solution:

According to the question,
$5\cos^2\theta + 7\sin^2\theta – 6 = 0$
We know that,
$\sin^2\theta = 1 – \cos^2\theta$
Therefore, $5\cos^2\theta + 7(1 – \cos^2\theta) – 6 = 0$
$\Rightarrow 5\cos^2\theta + 7 – 7\cos^2\theta – 6 = 0$
$\Rightarrow -2\cos^2\theta + 1 = 0$
$\Rightarrow \cos^2\theta = ½$
Therefore, $\cos\theta = \pm 1/\sqrt{2}$
Therefore, $\cos\theta = \cos \pi/4$ or $\cos\theta = \cos 3\pi/4$
Since, solution of $\cos x = \cos\alpha$ is given by
$x = 2m\pi \pm \alpha \ \forall\ m \in Z$
$\theta = n\pi \pm \pi/4, n \in Z$

28. Find the general solution of the equation sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x
Solution:

According to the question,
sin x – 3sin2x + sin3x = cos x – 3cos2x + cos3x
Grouping sin x and sin 3x in LHS and, cos x and cos 3x in RHS,
We get,
sin x + sin3x – 3sin2x = cos x + cos3x – 3cos2x
Applying transformation formula,
cos A + cos B = 2cos ((A + B)/2) cos((A – B)/2)

$\sin A + \sin B = 2\sin((A + B)/2)\cos((A – B)/2)$

$\Rightarrow 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - 3\cos 2x$

⇒ 2sin 2x cos x – 3sin 2x = 2cos 2x cos x – 3cos 2x

⇒ 2sin 2x cos x – 3sin 2x – 2cos 2x cos x + 3cos 2x = 0

⇒ 2cos x (sin 2x – cos 2x) – 3(sin 2x – cos 2x) = 0

⇒ (sin 2x – cos 2x)(2cos x – 3) = 0

⇒ cos x = 3/2 or sin 2x = cos 2x

As cos x ∈ [-1,1]

Hence, no value of x exists for which cos x = 3/2

Therefore, sin 2x = cos 2x

⇒ tan 2x = 1 = tan π/4

We know solution of tan x = tan α is given by,

x= nπ ± α , n ∈ Z

Therefore, 2x = nπ ± (π/4)

⇒ x = nπ/2 ± (π/8), n ∈ Z

29. Find the general solution of the equation (√3 – 1) cos θ + (√3 + 1) sin θ = 2
[Hint: Put √3 – 1 = r sin α, √3 + 1 = r cos α which gives tan α = tan((π/4) – (π/6)) α = π/12]
Solution:

Let, r sinα = √3 – 1 and r cosα = √3 + 1

Therefore, $r = \sqrt{\{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2\}} = \sqrt{8} = 2\sqrt{2}$

And, tan α = (√3 – 1) / (√3 + 1)

Therefore, r(sinα cos θ + cosα sin θ) = 2

⇒ r sin (θ+α) = 2

⇒ sin (θ+α) = 1/√2

⇒ sin (θ+α) = sin (π/4)

⇒ $\theta+\alpha = n\pi + (-1)^n (\pi/4)$, n ∈ Z

⇒ $\theta = n\pi + (-1)^n (\pi/4) - (\pi/12)$, n ∈ Z

OBJECTIVE TYPE QUESTIONS

30. If sin θ + cosec θ = 2, then $\sin^2\theta + \text{cosec}^2\theta$ is equal to
A. 1
B.4
C. 2
D. None of these
Solution:

C. 2

Explanation:

According to the question,

sin θ + cosec θ = 2

Squaring LHS and RHS,

We get,

⇒ $(\sin\theta + \text{cosec}\,\theta)^2 = 2^2$

⇒ $\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta = 4$

$\Rightarrow \sin^2\theta + \text{cosec}^2\theta + 2 \sin\theta\,(1/\sin)\theta = 4$
$\Rightarrow \sin^2\theta + \text{cosec}^2\theta + 2 = 4$
$\Rightarrow \sin^2\theta + \text{cosec}^2\theta = 2$
Thus, option (C) 2 is the correct answer.

31. If $f(x) = \cos^2x + \sec^2x$, then
A. $f(x) < 1$
B. $f(x) = 1$
C. $2 < f(x) < 1$
D. $f(x) \geq 2$
[Hint: A.M $\geq$ G.M.]
Solution:

D. $f(x) \geq 2$

Explanation:

According to the question,
We have, $f(x) = \cos^2x + \sec^2x$
We know that, A.M $\geq$ G.M.

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \sec^2 x}$$

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \frac{1}{\cos^2 x}}$$

$$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq 1$$

$\Rightarrow \cos0^2x + \sec^2x \geq 2$
$\Rightarrow f(x) \geq 2$
Thus, option (D) $f(x) \geq 2$ is the correct answer.

32. If $\tan\theta = 1/2$ and $\tan\phi = 1/3$, then the value of $\theta + \phi$ is
A. $\pi/6$
B. π
C. 0
D. $\pi/4$
Solution:

D. $\pi/4$

Explanation:

According to the question,

$\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

We know that,

$$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

$$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\Rightarrow \tan(\theta+\phi) = \tan\frac{\pi}{4}$$

Thus, option (D) $\pi/4$ is the correct answer.

33. Which of the following is not correct?

A. sin θ = – 1/5

B. cos θ = 1

C. sec θ = ½

D. tan θ = 20

Solution:

C. sec θ = ½

Explanation:

According to the question,

We know that,

a) sin θ = – 1/5 is correct since Sin θ ∈ [-1,1]

b) cos θ = 1 is correct since Cos θ ∈ [-1,1]

c) sec θ = ½

⇒ (1/cos θ) = ½

⇒cos θ=2 is incorrect since Cos θ ∈ [-1,1]

d) tan θ = 20 is correct since tan θ ∈ R.

Thus, option (C) sec θ = ½ is the correct answer.

34. The value of tan 1° tan 2° tan 3°… tan 89° is

A. 0

B. 1

C. ½

D. Not defined

Solution:

B. 1

Explanation:

According to the question,

tan 1° tan 2° tan 3°… tan 89°

= tan 1° tan 2° … tan 45° tan (90-44°) tan(90-43°)…tan (90-1°)

= tan 1°tan 2° … tan 45°cot 44°cot 43°…cot 1° [∵tan (90-θ)=cot θ]

= tan 1° cot 1° tan 2° cot 2°…tan45°… tan 89° cot 89°

=1.1….1 = 1

Thus, option (B) 1 is the correct answer.

35. The value of $(1 - \tan^2 15^o)/(1 + \tan^2 15^o)$ is
A. 1
B. √3
C. √3/2
D. 2
Solution:

C. √3/2

Explanation:

According to the question,

Let θ = 15° ⇒ 2θ = 30°

Now, since we know that,

$$\mathrm{Cos}2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\Rightarrow \cos30° = \frac{1-\tan^2 15°}{1+\tan^2 15°}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{1-\tan^2 15°}{1+\tan^2 15°}$$

Thus, option (C) √3/2 is the correct answer.

36. The value of cos 1° cos 2° cos 3°… cos 179° is
A. 1/√2
B. 0
C. 1
D. –1
Solution:

B. 0

Explanation:

According to the question,

Since cos90° =0

We get,

⇒ cos 1° cos 2° cos 3°… cos90°… cos 179° = 0

Thus, option (B) 0 is the correct answer.

37. If tan θ = 3 and θ lies in third quadrant, then the value of sin θ is
A. 1/√10
B. – 1/√10
C. – 3/√10
D. 3/√10
Solution:

C. – 3/√10

Explanation:

According to the question,

Given that, tan θ = 3 and θ lies in third quadrant

⇒ cot θ = 1/3

We know that,

$\text{Cosec}^2\theta = 1+\cot^2\theta$

$$= 1 + \left(\frac{1}{3}\right)^2 = 1 + \frac{1}{9} = \frac{10}{9}$$

$$\Rightarrow \sin^2\theta = \frac{9}{10}$$

$$\Rightarrow \sin\theta = \pm\frac{3}{\sqrt{10}}$$

$$\Rightarrow \sin\theta = -\frac{3}{\sqrt{10}}$$,since θ lies in third quadrant.

Thus, option (C) – $3/\sqrt{10}$ is the correct answer.

38. The value of tan 75°– cot 75° is equal to

A. $2\sqrt{3}$

B. $2 + \sqrt{3}$

C. $2 - \sqrt{3}$

D. 1

Solution:

A. $2\sqrt{3}$

<u>Explanation:</u>

According to the question,

We have,

$\tan 75° - \cot 75°$

$$= \frac{\sin 75°}{\cos 75°} - \frac{\cos 75°}{\sin 75°}$$

$$= \frac{\sin^2 75° - \cos^2 75°}{\cos 75° \sin 75°}$$

$$= \frac{2(\sin^2 75° - \cos^2 75°)}{2\cos 75° \sin 75°}$$

$$= \frac{-2\cos 150°}{\sin 150°}$$

$= -2\cot 150°$

$= -2\cot(180°-30°)$

$= 2\cot 30°$

$=2\sqrt{3}$

Thus, option (A) $2\sqrt{3}$ is the correct answer.

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Click Here to Join
SST

Click Here to Join
Mathematics

Click Here to Join
Science

Click Here to Join
English

Click Here to Join
Hindi-A

Click Here to Join
IT Code-402

Click Here to Join
Hindi-B

Click Here to Join
Artificial Intelligence

Senior Secondary Groups (XI & XII For Teachers Only)

Click Here to Join
Physics

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Chemistry

Click Here to Join
English

Click Here to Join
Mathematics

Click Here to Join
Biology

Click Here to Join
Accountancy

Click Here to Join
Economics

Click Here to Join
BST

Click Here to Join
History

Click Here to Join
Geography

Click Here to Join
Sociology

Click Here to Join
Hindi Elective

Click Here to Join
Hindi Core

Click Here to Join
Home Science

Click Here to Join
Sanskrit

Click Here to Join
Psychology

Click Here to Join
Political Science

Click Here to Join
Painting

Click Here to Join
Vocal Music

Click Here to Join
Comp. Science

Click Here to Join
IP

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Physical Education

Click Here to Join
APP. Mathematics

Click Here to Join
Legal Studies

Click Here to Join
Entrepreneurship

Click Here to Join
French

Click Here to Join
IT

Click Here to Join
Artificial Intelligence

Other Important Groups (For Teachers & Principal's)

Click Here to Join
Principal's Group

Click Here to Join
Teachers Jobs

Click Here to Join
IIT/NEET

Subject Wise Secondary and Senior Secondary Groups (IX & X For Students Only)

Secondary Groups (IX & X)

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SST

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Mathematics

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Science

Click Here to Join
English

Click Here to Join
Hindi

Click Here to Join
IT Code

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Artificial Intelligence

Senior Secondary Groups (XI & XII For Students Only)

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Music

Click Here to Join
Comp. Science

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Physical Education

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Legal Studies

Click Here to Join
Entrepreneurship

Click Here to Join
French

Click Here to Join
IT

Click Here to Join
AI

Click Here to Join
IIT/NEET

Click Here to Join
CUET

Groups Rules & Regulations:

To maximize the benefits of these WhatsApp groups, follow these guidelines:

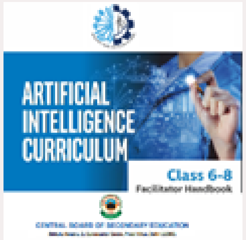
1. Share your valuable resources with the group.
2. Help your fellow educators by answering their queries.
3. Watch and engage with shared videos in the group.
4. Distribute WhatsApp group resources among your students.
5. Encourage your colleagues to join these groups.

Additional notes:

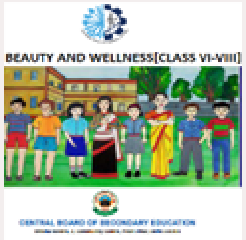
1. Avoid posting messages between 9 PM and 7 AM.
2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
   - No introductions.
   - No greetings or wish messages.
   - No personal chats or messages.
   - No spam. Or voice calls
   - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL

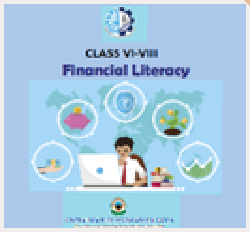


Artificial Intelligence



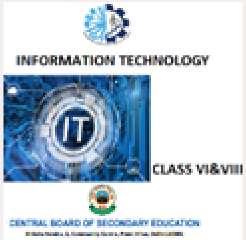
Beauty & Wellness

Design Thinking & Innovation



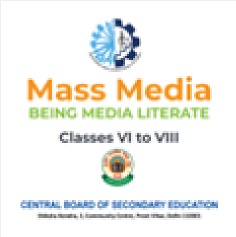
Financial Literacy

Handicrafts

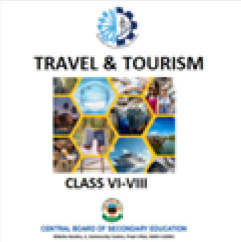


Information Technology

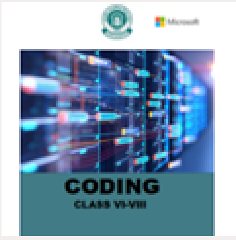
Marketing/Commercial Application



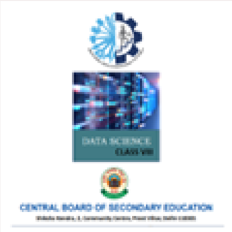
Mass Media - Being Media Literate



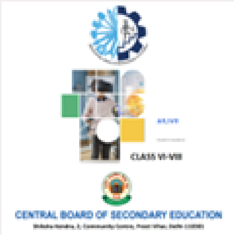
Travel & Tourism



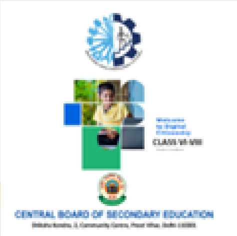
Coding



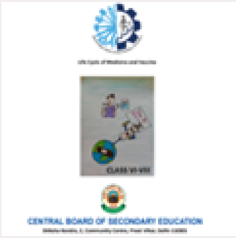
Data Science (Class VIII only)



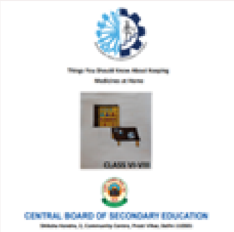
Augmented Reality / Virtual Reality



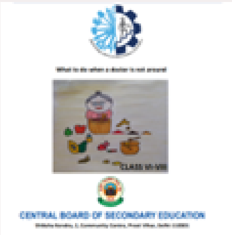
Digital Citizenship



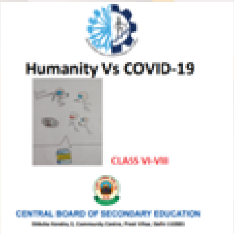
Life Cycle of Medicine & Vaccine



Things you should know about keeping Medicines at home

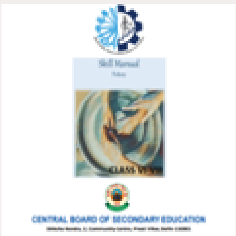


What to do when Doctor is not around

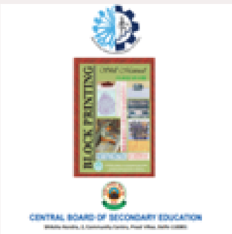


Humanity & Covid-19

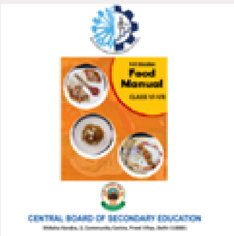
Blue Pottery



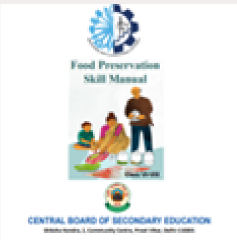
Pottery



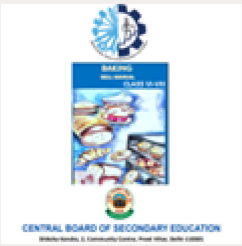
Block Printing



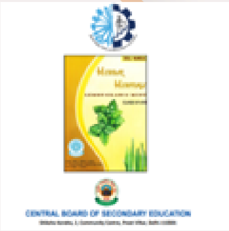
Food



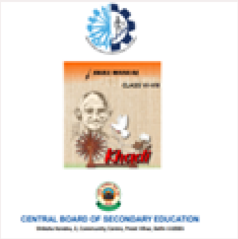
Food Preservation



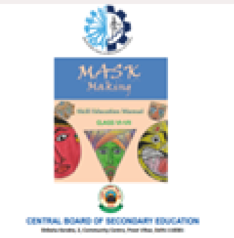
Baking



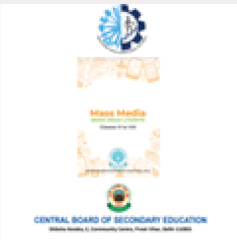
Herbal Heritage



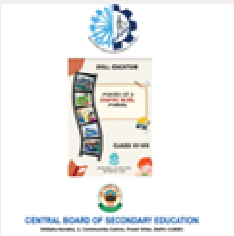
Khadi



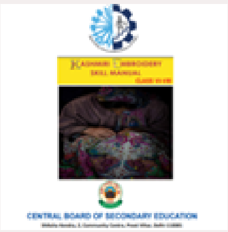
Mask Making



Mass Media

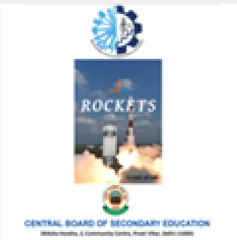


Making of a Graphic Novel

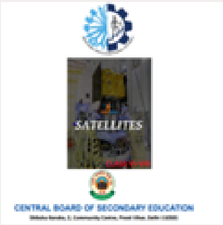


Kashmiri Embroidery

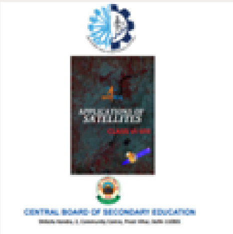
Embroidery



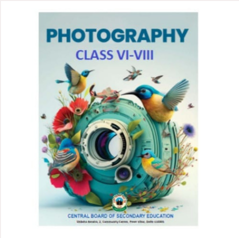
Rockets



Satellites

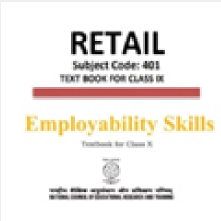


Application of Satellites



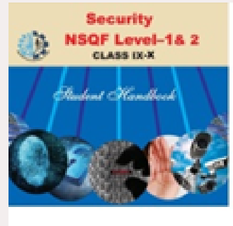
Photography

SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)

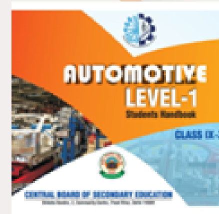


Retail

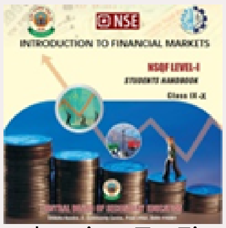
Information Technology



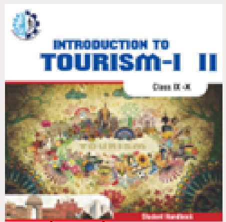
Security



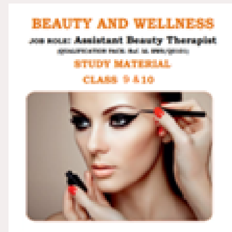
Automotive



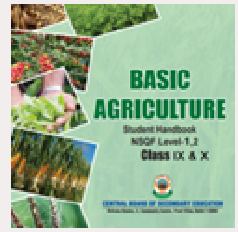
Introduction To Financial Markets



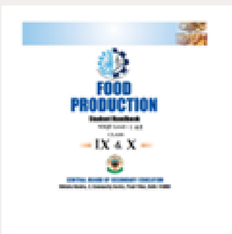
Introduction To Tourism



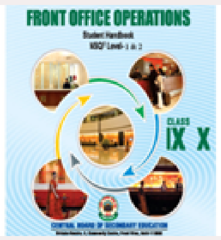
Beauty & Wellness



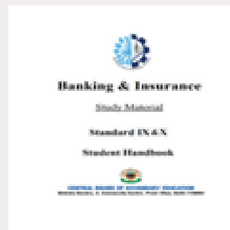
Agriculture



Food Production

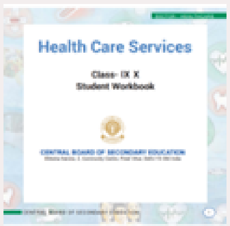


Front Office Operations



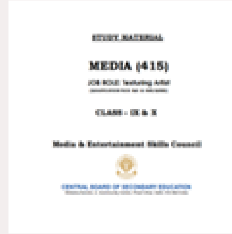
Banking & Insurance

Marketing & Sales

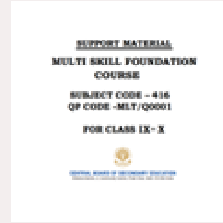


Health Care

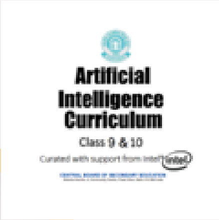
Apparel



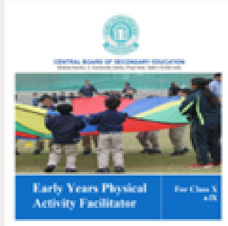
Multi Media



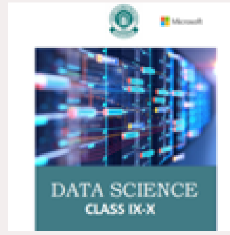
Multi Skill Foundation Course



Artificial Intelligence

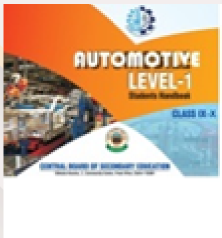


Physical Activity Trainer

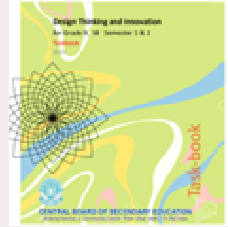


Data Science

Electronics & Hardware (NEW)

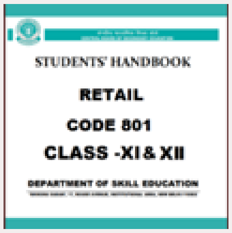


Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

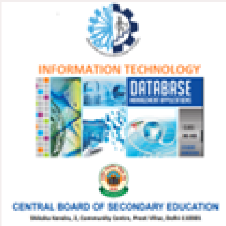


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)



Retail

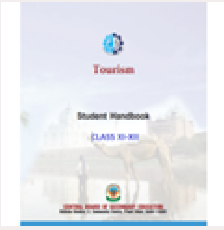


InformationTechnology

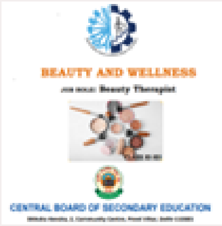
Web Application

Automotive

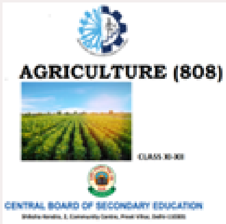
Financial Markets Management



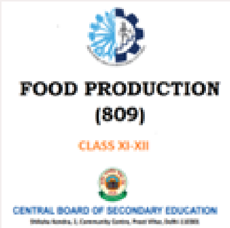
Tourism



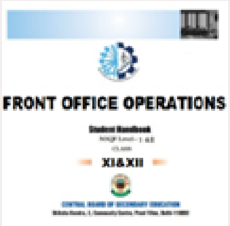
Beauty & Wellness



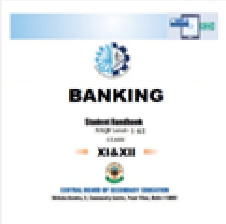
Agriculture



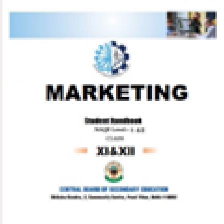
Food Production



Front Office Operations



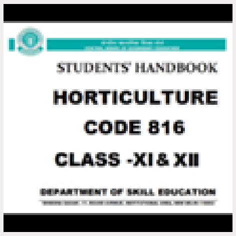
Banking



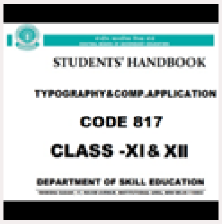
Marketing

Health Care

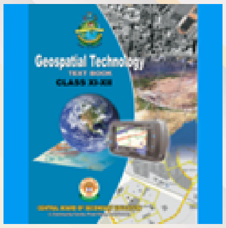
Insurance



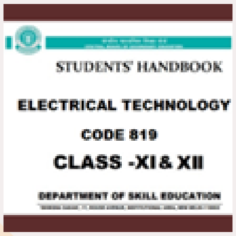
Horticulture



Typography & Comp. Application

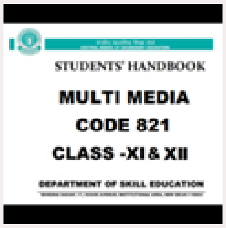


Geospatial Technology

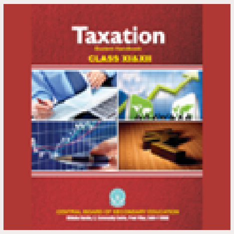


Electrical Technology

Electronic Technology

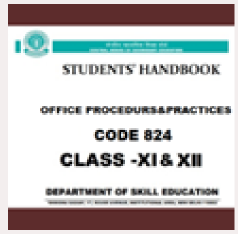


Multi-Media

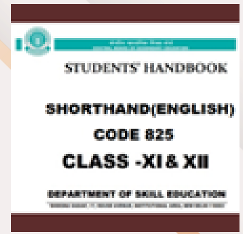


Taxation

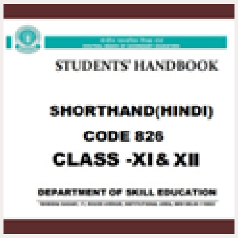
Cost Accounting



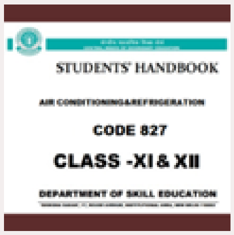
Office Procedures & Practices



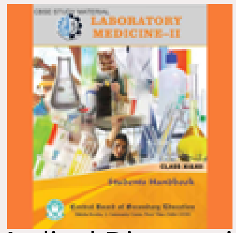
Shorthand (English)



Shorthand (Hindi)

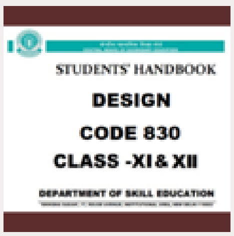


Air-Conditioning & Refrigeration

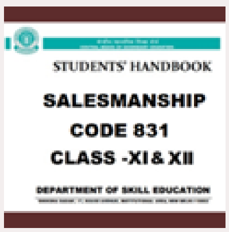


Medical Diagnostics

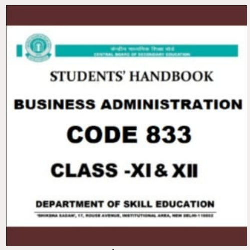
Textile Design



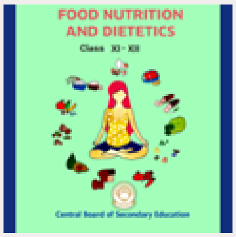
Design



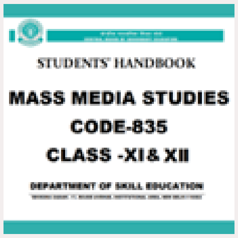
Salesmanship



Business Administration

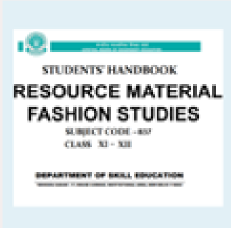


Food Nutrition & Dietetics

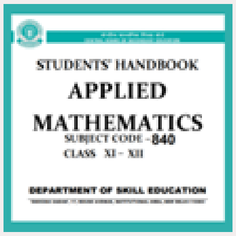


Mass Media Studies

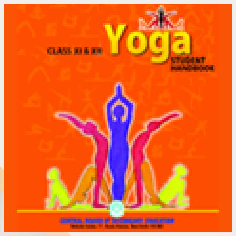
Library & Information Science



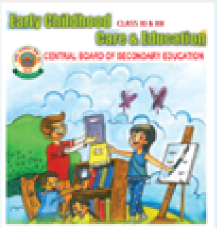
Fashion Studies



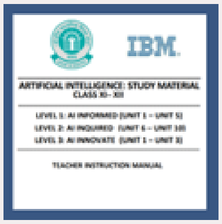
Applied Mathematics



Yoga

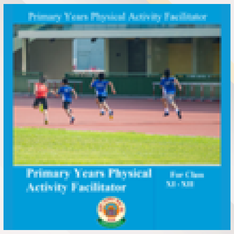


Early Childhood Care & Education

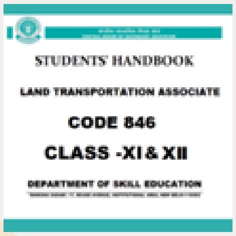


Artificial Intelligence

Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)

Electronics & Hardware (NEW)

Design Thinking & Innovation (NEW)